# Electromagnetically induced transparency in an atom-molecule Bose-Einstein condensate

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(February 6, 2008)

We propose a new measurement scheme for the atom-molecule dark state by using electromagnetically induced transparency (EIT) technique. Based on a density-matrix formalism, we calculate the absorption coefficient numerically. The appearance of the EIT dip in the spectra profile gives clear evidence for the creation of the dark state in the atom-molecule Bose-Einstein condensate.
PACS numbers: 03.75.Nt, 42.50.Gy, 32.80.Qk

The phenomenon of dark states is well known in quantum optics and is based on a superposition of long-lived system eigenstates which decouples from the light field. The coherent dark states lead to the phenomenon of electromagnetically induced transparency [1,2]. The light-matter coupling associated with the EIT can be used for the preparation and detection of coherent matter wave phenomena in ultra-cold quantum gases. For instance, EIT has been suggested as a probe for the diffusion of the relative phase in a two-component Bose Einstein condensate (BEC) [3]. The absorption apectra of the EIT in $\Lambda$ configuration BEC have also been studied both experimentally [4,5] and theoretically [6,7]. Hau et al., have demonstrated the reduction of the group velocity of a light pulse to 17 m/s [8]. Using similar experimental setup, Liu et al., demonstrated the coherent storage and read out of optical information in the ultracold EIT sample [9].

Recently, possibility of preparing atom-molecule Bose-Einstein condensate (AMBEC) has attracted wide attentions [10–13]. Condensed bosonic atoms can be converted to a molecular condensate by using either the photoassociation process (PA) [10,14,15] or the so-called Feshbach resonance method [11,12,16,17]. In one-color PA, two colliding atoms absorb a photon and form an electronically excited molecule. To overcome the generation of the highly excited molecules, two-color free-bound-bound mechanism, i.e., stimulated Raman adiabatic passage was proposed to convert atomic BEC into a molecular condensate [18–24]. The two-color PA scheme relies on the atom-molecule dark state (AMDS). Besides the dynamical property of the PA, photoassociative spectroscopy of the one-color PA [25,26] and the two-color PA [27,28] has been studied intensively in the ultracold atom-molecule system. Most recently, Winkler et al., demonstrated the creation of the AMDS by measuring two-color PA spectra [29]. Dumke et al., observed similar spectrum profile in the thermal sodium gas [30]. The PA spectra measurement is in fact the population of atoms or excited molecules (i.e., diagonal density matrix elements), which does not imply the macroscopic coherence between atoms and molecules [30].

In this Letter, we propose an absorption imagining study of electromagnetically induced transparency in a $\Lambda$-type atom-molecule Bose-Einstein condensate. Our scheme has many advantages, such as (i) the absorption spectra in multilevel atomic systems has been well-understood in terms of the dark states; (ii) the observation time (several $\mu$s) of our scheme is much shorter than that of Ref. [29] (typical ten ms), which drastically reduces the influence of the particle losses. Our study is outlined as follows. First of all, we derive a density-matrix formalism from a coupled Gross-Pitaevski equations. Previous results in Ref. [29] are reproduced by calculating the atomic population. To proceed, we investigate temporal evolution of the off-diagonal density matrix elements. Our results show a destructive interference in the AMBEC. We focus on the absorption coefficient of weak-probe field, and find the EIT dip in the spectra, which exhibits the creation of the AMDS. Finally, we present analytically the linear susceptibility of the AMBEC.

We consider a $\Lambda$ configuration atom-molecular BEC system [29], where two ground states $|a\rangle$ and $|g\rangle$ are coupled to an intermediate excited state $|b\rangle$ by two external optical fields. In the framework of mean-field theory, the dynamics can be well described by the Gross-Pitaevski equations for the normalized field amplitudes $a$, $b$, and $g$ [18–24] ($\hbar = 1$):

$$\dot{a} = i\Omega_1 a^* b, \tag{1a}$$

$$\dot{b} = (i\Delta_1 - \gamma_b/2)\, b + \frac{i}{2}(\Omega_1 a^2 + \Omega_2 g), \tag{1b}$$

$$\dot{g} = (i\delta - \gamma_g/2)g - \frac{i}{2}\Omega_2 b, \tag{1c}$$

where $\Omega_j$ with $j = 1, 2$, are the free-bound and the bound-bound Rabi frequencies, respectively. Due to Bose-enhancement, $\Omega_1$ is proportional to $\sqrt{\varrho_0}$, where $\varrho_0$ is the atomic initial density. The decay rates $\gamma_b$ and $\gamma_g$ of the molecules are included phenomenologically. It was shown that $\gamma_g$ is intensity dependent [29]. The two-photon detuning is $\delta = \Delta_1 - \Delta_2$, where $\Delta_1 = \omega_1 - (E_b - 2E_a)$ and $\Delta_2 = \omega_2 - (E_b - E_g)$. Here, $\omega_j$ with $j = 1, 2$ are frequencies of the fields, and $E_\alpha$ with $\alpha = a, b, g$ are the internal energies of species $\alpha$, re-

spectively. The atom-atom, atom-molecule interactions contribute mean-field shifts to the detunings in Eq. (1). For low atomic density, e.g., $\varrho_0 = 2 \times 10^{14}\,\mathrm{cm}^{-3}$, the shifts are small and can be neglected.

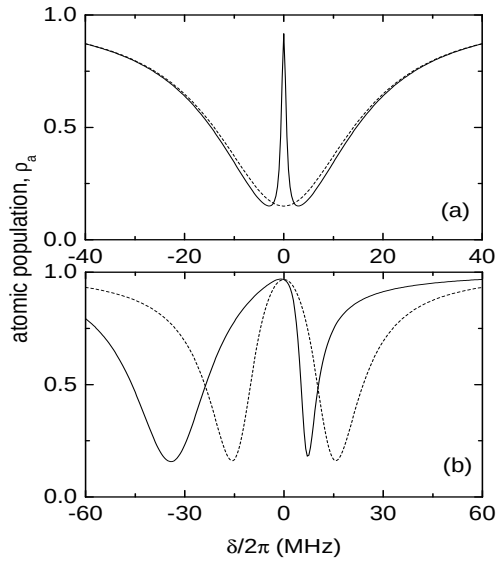


FIG. 1. Dark resonances in two-color photoassociation spectra for (a) the low power case $I_1 = 7\ \mathrm{W\,cm^{-2}}$, and $I_2 = 0$ (the dotted line), 0.7 $\mathrm{W\,cm^{-2}}$ (the solid line) . $\Delta_2 = 0$; (b) high power case $I_1 = 80\ \mathrm{W/cm^2}$ and $I_2 = 20\ \mathrm{W/cm^2}$. The detuning are $\Delta_2 = 0$ (the dotted line), $2\pi \times 27$ MHz (the solid line).

To investigate atom-molecule coherence, we further introduce $3 \times 3$ dimensional density matrix elements as the following [31]: $\rho_a = |a|^2$, $\rho_b = |b|^2$, $\rho_g = |g|^2$, $\rho_{ba} = b(a^*)^2$, $\rho_{ga} = g(a^*)^2$, and $\rho_{bg} = bg^*$, where $\rho_{ba}$ ($\rho_{ga}$) describes the photoassociation of two atoms into one excited (ground) molecule. Within a rotating frame, the equations of motion for the populations are

$$\dot{\rho}_a = i\Omega_1(\rho_{ba} - c.c), \tag{2a}$$

$$\dot{\rho}_g = -\gamma_g\rho_g + i\frac{\Omega_2}{2}(\rho_{bg} - c.c), \tag{2b}$$

$$\dot{\rho}_b = -\gamma_b\rho_b - i\frac{\Omega_1}{2}(\rho_{ba} - c.c) - i\frac{\Omega_2}{2}(\rho_{bg} - c.c). \tag{2c}$$

In the absence of molecular decaying, i.e., $\gamma_b = \gamma_g = 0$, the total atom number is conserved and $\rho_a + 2(\rho_b + \rho_g) = 1$. The coherence part density matrix elements obey

$$\dot{\rho}_{ga} = (i\delta - \frac{\gamma_g}{2})\rho_{ga} - 2i\Omega_1\rho_a\rho^*_{bg} + i\frac{\Omega_2}{2}\rho_{ba}, \tag{3a}$$

$$\dot{\rho}_{ba} = (i\Delta_1 - \frac{\gamma_b}{2})\rho_{ba} + i\frac{\Omega_1}{2}(\rho_a^2 - 4\rho_a\rho_b) + i\frac{\Omega_2}{2}\rho_{ga}, \tag{3b}$$

$$\dot{\rho}_{bg} = (i\Delta_2 - \frac{\Gamma}{2})\rho_{bg} + i\frac{\Omega_1}{2}\rho^*_{ga} + i\frac{\Omega_2}{2}(\rho_g - \rho_b), \tag{3c}$$

where $\Gamma = (\gamma_b + \gamma_g)$. Unlike the $\Lambda$-type atom system, the second terms of Eqs. (3a) and (3b) are nonlinear due to the free-bound transition.

We solve the above coupled differential equations (2) and (3) numerically using a fourth-order Runge-Kutta routine. Following Ref. [29], we study a dilute $^{87}$Rb BEC with a peak density $\varrho_0 = 2 \times 10^{14}\,\mathrm{cm}^{-3}$, then the parameters are $\gamma_b = 2\pi \times 13$ MHz, $\Omega_1/\sqrt{I_1} = 2\pi \times 8\,\mathrm{kHz}/(\mathrm{W\,cm^{-2}})^{1/2}$, and $\Omega_2/\sqrt{I_2} = 2\pi \times 7$ MHz$/(\mathrm{W\,cm^{-2}})^{1/2}$. The decay rate of the ground-state molecules is $\gamma_g = 2\pi \times 6\,\mathrm{kHz}/(\,\mathrm{W\,cm^{-2}})I_1 + \gamma_{\mathrm{bg}}$, where the background decay rate $\gamma_{\mathrm{bg}} \approx 2\pi \times 1\,\mathrm{kHz}$ at the peak density $\varrho_0$.

In Fig. 1(a), we consider the low-power field with $I_1 = 7\ \mathrm{W\,cm^{-2}}$, and $I_2 = 0$ (the dotted line), 0.7 $\mathrm{W\,cm^{-2}}$ (the solid line), respectively. Our results agree with the experimental results as expected [29]. In one-color photoassociation, the spectrum for a dipole-allowed transition is a Lorentzian with a width set by the optical transition linewidth. The presence of the coupling laser field results in the dark resonance appeared near the two-photon resonant point $\delta = 0$. The linewidth is much narrower than the natural linewidth of the excited molecules $\gamma_b$. In Fig. 1(b), we calculate $\rho_a(\delta)$ for relatively high laser power $I_1 = 80\ \mathrm{W/cm^2}$ and $I_2 = 20\ \mathrm{W/cm^2}$ with the detunings $\Delta_2 = 0$ (the dotted line), $2\pi \times 27$ MHz (the solid line). The results show relative broadened dark resonance lines [29], and nonzero $\Delta_2$ results in asymmetric spectrum, as shown by the solid line in Fig. 1(b).

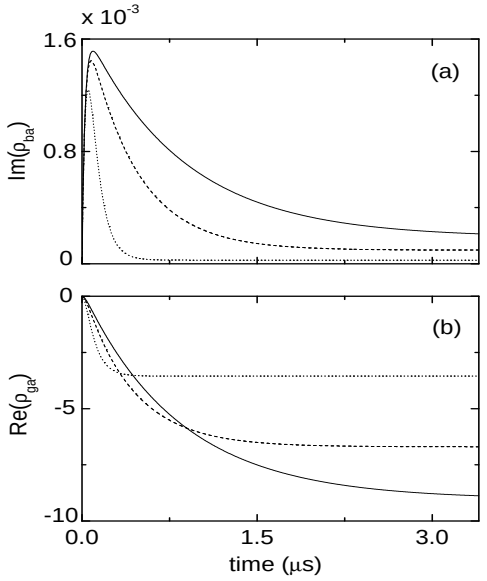


FIG. 2. Time evolution of $\mathrm{Im}(\rho_{ba})$ and $\mathrm{Re}(\rho_{ga})$ for $I_1 = 7\ \mathrm{W\,cm^{-2}}$, and $I_2 = 0.7\ \mathrm{W\,cm^{-2}}$ (the dotted lines), 0.18 $\mathrm{W\,cm^{-2}}$ (the dashed lines), and $I_1/80$ (the solid lines). $\Delta_1 = \Delta_2 = 0$.

The diagonal matrix element $\rho_a$ does not imply a coherent superposition of atoms and molecules [30]. To investigate the macroscopic coherence, we propose to measure the probe-field absorption coefficient, which is proportional to $\mathrm{Im}(\rho_{ba})$. Fig. 2 shows temporal evolution

of $\mathrm{Im}(\rho_{ba})$ and $\mathrm{Re}(\rho_{ga})$ for low-power case. We find that $\mathrm{Im}(\rho_{ba})$ grows quickly then decays to the steady-state values around $3.4\mu s$. The steady-state value of $\mathrm{Im}(\rho_{ba})$ depends on the coupling field Rabi frequency $\Omega_2$. The larger $\Omega_2$ is, the smaller the value is. Fig. 2(b) shows that $\mathrm{Re}(\rho_{ga})$ is always negative during total evolution, which implies destructive interference between the two competitive transitions.

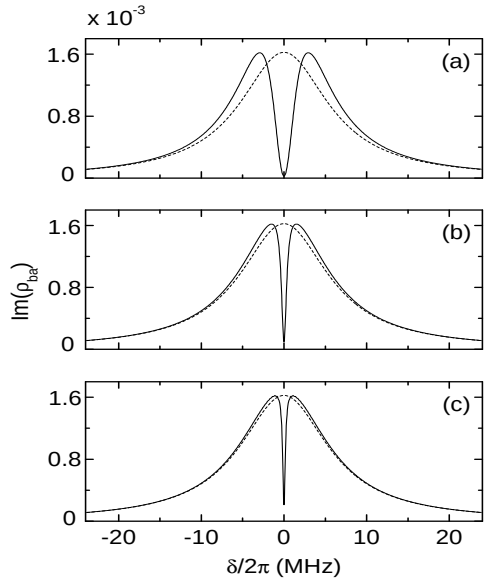


FIG. 3. Absorption spectra, $\mathrm{Im}(\rho_{ba})$ for $I_1 = 7\ \mathrm{W\,cm^{-2}}$, (a) $I_2 = 0.7\ \mathrm{W\,cm^{-2}}$, (b) $0.18\ \mathrm{W\,cm^{-2}}$, (c) $I_1/80$. $\Delta_2 = 0$. The dotted line is the case of $I_2 = 0$.

We calculate the absorption spectra of the probe field from Eqs. (2) and (3) for $I_1 = 7\ \mathrm{W\,cm^{-2}}$. In the absence of the coupling field, Lorentzian shape of $\mathrm{Im}(\rho_{ba})$ corresponds to the resonant absorption in the two-level system (see dotted lines in Fig. 3). For $\Omega_2 \neq 0$, the absorption spectrum exhibits an EIT dip near the resonant point $\delta = 0$. The depth and width of the dip are dependent on the coupling field Rabi frequency $\Omega_2$. For a relatively larger Rabi frequency, e.g., $I_2 = 0.7\ \mathrm{W\,cm^{-2}}$, the medium becomes almost transparent. With the decrease of $I_2$, the molecular losses become dominant, and the transparency is no longer perfect.

Time-evolution of $\mathrm{Im}(\rho_{ba})$ and $\mathrm{Re}(\rho_{ga})$ for the high-power cases with $I_1 = 80\ \mathrm{W\,cm^{-2}}$ and $I_2 = 20\ \mathrm{W\,cm^{-2}}$ exhibit damped oscillations, which tend to steady-state values at about $1\mu s$. Again $\mathrm{Re}(\rho_{ga}) < 0$ during time-evolution indicates the destructive interference in the high-power regions. In Fig. 4 (a), we calculate the absorption spectrum for $\Delta_2 = 0$. The absorption profile shows the signatures of an Autler-Townes doublet. The separation of two peaks are about $2\pi \times 30$ MHz, which agrees with the magnitude of Rabi frequency $\Omega_2$. For nonzero $\Delta_2$, $\mathrm{Im}(\rho_{ba})$ oscillates irregularly with time evolution, and the spectra become asymmetric and exhibit a board and narrow peak, as shown in Fig. 4(b) and 4(c).

According to Ref. [22], the Rabi frequencies take the forms of $\Omega_1 = d_1\mathcal{E}_1\mathcal{I}^{(1)}\sqrt{\varrho_0/2}$ and $\Omega_2 = d_2\mathcal{E}_2\mathcal{I}^{(2)}$, where $d_j$ $(j = 1, 2)$ are mean dipole matrix moments, $\mathcal{E}_j$ the slowly varying field amplitudes, and $\mathcal{I}^{(j)}$ the Franck-Condon overlap integrals, respectively. A factor of $\sqrt{2}$ is introduced in $\Omega_1$ to consistent with the notation in Ref. [22]. The linear susceptibility is $\chi^{(1)} = \frac{2\varrho_0 d_1^*}{\epsilon_0\mathcal{E}_1}\rho_{ba}^{(1)}$, where $\rho_{ba}^{(1)}$ is the first-order solution of $\rho_{ba}$. Within the weak-probe regime, i.e., $\Omega_1 \ll \Omega_2$, $\gamma_b$, the absorption of the AMBEC can be well-described by the first-order steady-state solution $\rho_{ba}^{(1)} = i\Omega_1/(2F)$, where $F = \gamma_b/2 - i\Delta_1 + \frac{\Omega_2^2/4}{\gamma_g/2 - i\delta}$ [32]. Combining the above discussions, we obtain the linear susceptibility of the AMBEC system

$$\chi^{(1)} = \frac{i\varrho_0^{3/2}|d_1|^2\mathcal{I}^{(1)}}{\sqrt{2}\epsilon_0 F}. \tag{4}$$

For $^{87}$Rb BEC, $\mathcal{I}^{(1)} \simeq 10^{-11}\mathrm{cm}^{3/2}$ and $\mathcal{I}^{(2)} \simeq 0.1$ [15,22]. Considering the density $\varrho_0 = 2 \times 10^{14}\mathrm{cm}^{-3}$ and the probe field intensity $I_1 = 7\mathrm{Wcm}^{-2}$, we obtain $\mathcal{E}_1 = \sqrt{2I_1/(c\epsilon_0)} = 7.264 \times 10^3\mathrm{V/m}$. Consequently, we can calculate the mean dipole matrix moment $d_1 = 19.3 \times 10^{-30}\mathrm{Cm}$. It was shown by Ref. [26], the dipole matrix moment of the free-bound transition to the excited molecular state is relatively smaller than the atomic dipole.

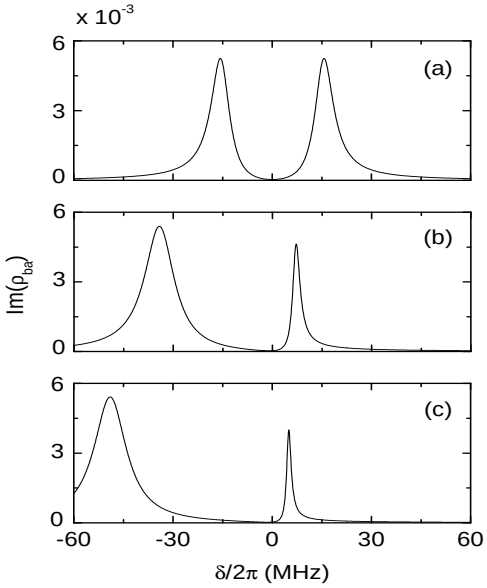


FIG. 4. Absorption spectra, $\mathrm{Im}(\rho_{ba})$ for $I_1 = 80\ \mathrm{W\,cm^{-2}}$, $I_2 = 20\ \mathrm{W\,cm^{-2}}$. (a) $\Delta_2 = 0$, (b) $\Delta_2 = 2\pi \times 27$ (MHz), and (c) $\Delta_2 = 2\pi \times 44$ (MHz).

In summary, we have proposed the possibility of observing the EIT in the AMBEC system. From the introduced density matrix formalism, we study how the transparency of a probe beam of free-bound transition can be

controlled via a coupling light beam resonant with the bound-bound transition. All of our results are consistent with that of Ref. [29]. We show that in weak-probe limit, the absorption profile can be well-described by the first-order steady-state solution of the matrix element. We present analytical expression of the linear susceptibility $\chi^{(1)}$, and calculate electric dipole moment, $d_1$, for the free-bound transition. Finally, we would like to emphasize that, compared with the previous photoassociative spectroscopy [29], the measurement of probe-field absorption has its advantage to give direct coherence information of the AMBEC dark state, and the observation time of our scheme is much shorter so that the particle dissipation effects can be reduced significantly.

This work is supported in part by the BK21 and by KOSEF through Center for Strongly Correlated Materials Research, SNU (2005).

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